Interactions in nonlinear analysis: variational methods, reaction-diffusion equations, dynamical systems and applications

EELISA Innocore Connect Workshop
Universidad Politécnica de Madrid
Escuela Técnica Superior de Ingeniería y Diseño Industrial

April 10th–12th 2024

Updated information will be available at this link

Organizers

- Makrina Agaoglou (UPM)
- Emeric Bouin (PSL)
- Pedro González Manchón (UPM)
- Andrea Malchiodi (SNS)
- Andrea Tellini (UPM)

Schedule

Time	Wednesday, April 10th (Sala Azul – Blue room – Floor -1)	Thursday, April 11th (Salón de actos – Conference hall – Floor 2)	Friday, April 12th (Sala Azul – Blue room – Floor -1)
9:30-10:00	Welcome session		Fabricio Macià Lang Universidad Politécnica de Madrid
10:00-10:30	Andrea Malchiodi Scuola Normale Superiore	François Hamel Aix-Marseille Université	
10:30-11:00			Andrea Tellini Universidad Politécnica de Madrid
11:00-11:30	María del Mar González Universidad Autónoma de Madrid	Juan Carlos Sampedro Universidad Politécnica de Madrid	
11:30-12:00			*Break*
12:00-12:30	*Break*	*Break*	Luca Ziviani Université Paris Sciences et Lettres
12:30-13:00	Francesco Malizia Scuola Normale Superiore	Mattia Freguglia Scuola Normale Superiore	
13:00-13:30			Eduardo Muñoz-Hernández Universidad Complutense de Madrid
13:30-14:00	*Break*	*Break*	
14:00-15:00			
15:00-16:00	Miguel A. F. Sanjuán Universidad Rey Juan Carlos	Idriss Mazari-Fouquer Université Paris Sciences et Lettres	
16:00-16:30	*Break*	*Break*	
16:30-17:30	Michele Caselli Scuola Normale Superiore	Makrina Agaoglou Universidad Politécnica de Madrid	

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Makrina Agaoglou
Universidad Politécnica de Madrid

The method of Lagrangian descriptors: applications in geoscience and in chemical reactions

Abstract

In this talk, I will present the method of Lagrangian descriptors explaining the transport in a chemical reaction and also in oceanic models. I will also present a new uncertainty quantification measure appropriate for quantifying the performance of models in assessing for example the origin or source of a given observation. We have noticed that in a neighborhood of the observation this uncertainty measure is related to the invariant dynamical structures of the model such as hyperbolic trajectories and their stable and unstable manifolds.

Michele Caselli
Scuola Normale Superiore

Classification of finite index solutions to the fractional Allen-Cahn equation and geometric consequences

Abstract

In this talk, I will describe a recent result, jointly with Enric Florit-Simon and Joaquim Serra, on the finite index version of the fractional De Giorgi (and Bernstein) conjecture in dimension 3 and 4. Then, we will deduce some geometric consequences regarding the fractional Allen-Cahn equation on Riemannian manifolds.

Mattia Freguglia
Scuola Normale Superiore

Phase-field approximation of the Willmore functional and a conjecture of De Giorgi

Abstract

In recent years, there has been a growing interest in geometric energies, as for example the Area functional or the Willmore functional. Some interesting problems arise in the study of the associated geometric flows or in proving the existence of critical points of suitable types. However, these questions are usually difficult to answer, and sometimes it is useful to approximate the original functionals with simpler ones.

In 1991, De Giorgi conjectured a possible approximation of the Willmore functional, in the sense of Gamma-convergence, based on the first variation of the Allen-Cahn functionals.

Since then, several authors have investigated this problem, and some modifications of the approximating functionals have been considered. For some of these, it has been proven that they actually approximate the Willmore functional on smooth sets.

After reviewing some of the most relevant contributions originated from this conjecture over time, we will present a result obtained in collaboration with G. Bellettini and N. Picenni, where we provide a negative answer to the original conjecture.

María del Mar González Nogueras
Universidad Autónoma de Madrid

Spectral properties of Levy Fokker-Planck equations

Abstract

We study the spectrum of a fractional Laplacian equation with drift. This operator arises when studying the fractional heat equation in self-similar variables. We find, in the radially symmetric case, a suitable functional space framework such that all the spectrum reduces to eigenvalues, and we calculate precisely all the eigenfunctions in terms of Laguerre polynomials. The proofs involve conformal geometry, Mellin transform and complex analysis methods.

François Hamel
Université Aix-Marseille

Large-time dynamics of solutions of reaction-diffusion equations in $\mathbb{R}^N$ with general initial support

Abstract

The talk will focus on the large-time dynamics of bounded solutions of reaction-diffusion equations in $\mathbb{R}^N$ with general bounded or unbounded initial support. I will discuss the existence of spreading speeds and spreading sets of the solutions in any direction, in connection with the existence of planar traveling fronts. I will also explain some results on the asymptotic one-dimensional symmetry of the elements of the Ω-limit sets of the solutions. Lastly, I plan to discuss the influence of the fragmentation of the initial support on the large-time dynamics.

The talk is based on joint works with Matthieu Alfaro, Lionel Roques, and Luca Rossi.

Fabricio Macià Lang
Universidad Politécnica de Madrid

Solving ill-posed inverse problems via the Born approximation

Abstract

In this talk we discuss recent work on inverse problems for elliptic partial differential equations revolving around the concept of Born approximation, a tool originally introduced in the context of Scattering Theory that has been extensively used in the physics and computational literature. We will focus on the inverse problem of recovering the potential of a Schrödinger operator from the knowledge of its Dirichlet-to-Neumann map. This is known as the Calderón problem, or Electric Impedance Tomography, and has been intensively studied in the past forty years. This inverse problem is severely ill-posed, which makes the task of designing efficient algorithms to solve it particularly difficult. We will rigorously prove, in the simplified setting of radial potentials in the euclidean unit ball, the existence of the Born approximation, a function that encodes the whole DtN map and enjoys several interesting qualitative and quantitative approximation properties. We use this function to factorize the inverse problem into a linear (ill-posed but explicit) and a nonlinear (well-posed, Hölder continuous) part. This factorization gives a (partial) characterization of the set of DtN maps and we will show how this can be used to ultimately design efficient algorithms to solve the inverse problem. Our analysis is based on results on inverse spectral theory for Schrödinger operators on the half-line, in particular on the concept of A-amplitude introduced by Barry Simon in 1999. This talk is based on joint works with Juan Antonio Barceló, Carlos Castro, and Cristóbal Meroño (UPM), Thierry Daudé (Besançon), François Nicoleau (Nantes).

Andrea Malchiodi
Scuola Normale Superiore

Yamabe metrics on conical manifolds

Abstract

We prove existence of Yamabe metrics on singular manifolds with conical points and conical links of Einstein type that include orbifold structures. We deal with metrics of generic type and derive a counterpart of Aubin's classical result. Interestingly, the singular nature of the metric determines a different condition on the dimension, compared to the regular case. We derive asymptotic expansions on the Yamabe quotient by adding a proper and implicit lower-order correction to standard bubbles, whose contribution to the expansion of the quotient can be determined combining the decomposition of symmetric 2-tensor fields and Fourier analysis on the conical links.

This is joint work with M. Freguglia.

Francesco Malizia
Scuola Normale Superiore

Compactness of certain Palais-Smale sequences for a Liouville-type equation

Abstract

This talk will address the study of compactness for Palais-Smale sequences relative to a mean-field equation of Liouville type, that is, a 2-dimensional equation with exponential nonlinearity.

While the blow-up analysis for sequences of exact solutions is now well-known, the behaviour of general Palais-Smale sequences is still unknown. In this direction, I will present a result showing that if one considers Palais-Smale sequences with an additional asymptotic control on the Morse index, then it is possible to prove a concentration-compactness result analogous to the one known for exact solutions.

This is part of an ongoing project with Andrea Malchiodi.

Idriss Mazari-Fouquer
Université Paris Sciences et Lettres

Spatial ecology and game theory

Abstract

In this talk, we will discuss two recent works in collaboration with Z. Kobeissi and D. Ruiz-Balet, dealing with game-theoretical issues in population dynamics. The overall question we are seeking to address is: assuming that a population of fish lives in a domain, what is the influence of fishing on this population? Is it possible to quantify the fact that fishing is harmful? What happens if we assume that fishers compete and act rationally? To shed some light on this issue, we'll focus on two main models: 1) In the first, a finite number of agents attempt to find an equilibrium. We'll discuss some results on the existence of Nash equilibria, and present some qualitative properties. 2) In the second, an infinite number of agents are competing, and the formalism adopted is that of mean-field games. In this case, the question is that of the tragedy of the commons: is it possible for a species of fish that is invasive in the absence of fishermen to become extinct if the latter act in such a way as to optimize what they catch? And can collaboration between fishermen not only remedy this, but also enable each fisherman to obtain more fish in the end?

Eduardo Muñoz-Hernández
Universidad Complutense de Madrid

Topological tools for the analysis of BVPs and periodic problems

Abstract

During the last fifty years, a wide range of topological methods have been developed and sharpened for the analysis of differential equations. While for boundary value problems bifurcation theory has turned out to be a very important tool, for periodic problems a more classical approach has been the use of fixed point theorems and topological indices, e. g. the Poincaré-Birkhoff theorem and the Conley-Zehnder index, respectively, in order to get existence and multiplicity of periodic solutions. In this talk, we present different BVPs and periodic problems in which the different topological approaches above-cited have been applied. This is a joint work with J. López-Gómez (Universidad Complutense de Madrid), A. Boscaggin (Università di Torino) and F. Zanolin (Università di Udine).

Juan Carlos Sampedro
Universidad Politécnica de Madrid

A priori bounds for superlinear indefinite elliptic PDE in divergence form

Abstract

In this talk we present some new blow-up estimates for the positive explosive solutions of a paradigmatic class of elliptic boundary value problems of superlinear indefinite type:

$$\begin{cases} \mathscr{L}u = \lambda u + a(x)u^r & \text{in } \Omega, \\ \mathscr{B}u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain of $\mathbb{R}^N$, $N \geq 1$, of class $\mathcal{C}^2$, $\lambda \geq 0$,

$$\mathscr{L}u = -\mathrm{div}(A(x)\nabla u),$$

uniformly elliptic in Ω and $\mathscr{B}$ is any boundary operator of non-classical mixed type on $\partial\Omega$. These estimates are obtained by combining the scaling technique of Gidas–Spruck [2] together with a generalized De Giorgi–Moser weak Harnack inequality found, very recently, by Sirakov [4, 5]. In a further step, based on a comparison result of Amann and López-Gómez [1], we will show how these bounds provide us with some sharp a priori estimates for the classical positive solutions of (1). It turns out that this is the first general result where the decay rates of the potential $a(x)$ do not play any role for getting a priori bounds for the positive solutions when $N \geq 3$. This is a joint work with J. López-Gómez [3].

References

[1] H. Amann and J. López-Gómez, A priori bounds and multiple solutions for superlinear indefinite elliptic problems, *J. Diff. Equ.* **146**, (1998), 336–374.

[2] B. Gidas and J. Spruck, A priori bounds for positive solutions of nonlinear elliptic equations, *Comm. in Part. Diff. Equ.* **6** (1981), 883–901.

[3] J. López-Gómez and J. C. Sampedro, Blow-up estimates and a priori bounds for the positive solutions of a class of superlinear indefinite elliptic problems, *Submitted*, ArXiv: https://arxiv.org/abs/2402.01519.

[4] B. Sirakov, A new method of proving a priori bounds for superlinear elliptic PDE, *J. Math. Pures et Appl.* **141** (2020), 184–194.

[5] B. Sirakov, Global integrability and weak Harnack estimates for elliptic PDEs in divergence form, *Analysis & PDE* **15** (2022), 197–216.

Miguel A. F. Sanjuán
Universidad Rey Juan Carlos

Exploring the Unpredictability of Physical Systems: Basin Entropy and Testing for Wada Basins

Abstract

In nonlinear dynamics, basins of attraction are defined as the set of points that, taken as initial conditions, lead the system to a specific attractor. This notion appears in a broad range of applications where multistability is present, which is a common situation in neuroscience, economy, astronomy, ecology, and other disciplines. Nonlinear systems often give rise to fractal boundaries in phase space, hindering predictability. When a single boundary separates three or more different basins of attraction, we call them Wada basins. Usually, Wada basins have been considered even more unpredictable than fractal basins. However, this particular unpredictability has not been fully unveiled until the introduction of the concept of basin entropy. The basin entropy provides a quantitative measure of how unpredictable a basin is. With the help of several paradigmatic dynamical systems, we illustrate how to identify the ingredients that hinder the prediction of the final state. The basin entropy together with two new tests of the Wada property have been applied to some physical systems such as experiments of chaotic scattering of cold atoms, models of shadows of binary black holes, and classical and relativistic chaotic scattering associated to the Hénon-Heiles Hamiltonian system in astrophysics.

References

[1] A. Daza, A. Wagemakers, M.A.F. Sanjuán, and J. A. Yorke. Testing for Basins of Wada. Scientific Reports 5, 16579 (2015)

[2] A. Daza, A. Wagemakers, M.A.F. Sanjuán. Wada Property in Systems with Delay. CNSNS 43, 220–226 (2017)

[3] A. Daza, A. Wagemakers, B. Georgeot, D. Guéry-Odelin, and M.A.F. Sanjuán. Basin entropy: a new tool to analyze uncertainty in dynamical systems. Scientific Reports 6, 31416 (2016)

[4] A. Daza, B. Georgeot, D. Guéry-Odelin, A. Wagemakers, and M.A.F. Sanjuán. Chaotic dynamics and fractal structures in experiments with cold atoms. Phys. Rev. A 95, 013629 (2017)

[5] A. Daza, A. Wagemakers, B. Georgeot, D. Guéry-Odelin, and M.A.F. Sanjuán. Basin entropy, a measure of final state unpredictability and its application to the chaotic scattering of cold atoms. In Chaotic, Fractional, and Complex Dynamics: New Insights and Perspectives. Mark Edelman, Elbert Macau and Miguel A. F. Sanjuán, Editors. Springer, Cham, 2018

[6] Alvar Daza, Jake O. Shipley, Sam R. Dolan and Miguel A. F. Sanjuan. Wada structures in a binary black hole system. Phys. Rev. D 98, 084050 (2018)

[7] A. Daza, A. Wagemakers, M.A.F. Sanjuán. Ascertaining when a basin is Wada: the merging method. Scientific Reports 8, 9954 (2018)

[8] A. Wagemakers, A. Daza, M.A.F. Sanjuán. The saddle-straddle method to test for Wada basins CNSNS 84, 105167 (2020)

[9] A. Wagemakers, A. Daza, M.A.F. Sanjuán. How to detect Wada Basins. Discrete and Continuous Dynamical Systems B 26(1), 717-739 (2021)

[10] A. Puy, A. Daza, A. Wagemakers, M.A.F. Sanjuán. A test for fractal boundaries based on the basin entropy. Commun Nonlinear Sci Numer Simulat, 95, 105588 (2021)

[11] A. Daza, A. Wagemakers, M.A.F. Sanjuán. Classifying basins of attraction using the basin entropy. Chaos, Solitons & Fractals 159, 112112 (2022)

[12] A. Daza, A. Wagemakers, M.A.F. Sanjuán. Unpredictability and basin entropy. EPL, 141 43001 (2023)

Andrea Tellini
Universidad Politécnica de Madrid

Multiplicity of nodal steady-states for classical logistic equations

Abstract

Using as a base the result on multiplicity of 1-node solutions for degenerate logistic equations by López-Gómez and Rabinowitz (2020), I will first show how multiplicity also occurs for classical logistic equations, for weights which are small perturbations of the degenerate case. Then, I will show how multiplicity is valid far beyond this perturbative case, even arriving at cases arbitrarily close to situations where there is uniqueness.

These results are joint work with P. Cubillos and J. López-Gómez (Universidad Complutense de Madrid).

Luca Ziviani
Université Paris Sciences et Lettres

L^2 Hypocoercivity methods for kinetic Fokker-Planck equations with factorised Gibbs states

Abstract

This contribution deals with L^2 hypocoercivity methods for kinetic Fokker-Planck equations with integrable local equilibria and a *factorisation* property that relates the Fokker-Planck and the transport operators. Rates of convergence in presence of a global equilibrium, or decay rates otherwise, are estimated either by the corresponding rates in the diffusion limit, or by the rates of convergence to local equilibria, under moment conditions. On the basis of the underlying functional inequalities, we establish a classification of decay and convergence rates for large times, which includes for instance sub-exponential local equilibria and sub-exponential potentials.

This contribution is based on joint work with Emeric Bouin and Jean Dolbeault.